Erratum

1 Introduction

In the displayed equations between Theorems 7.5 and 7.6 there are a few - but not important - mistakes. First we do not have

$$\mathrm{Cov}(h_n(\omega_n), h_n(\omega_n^\varepsilon)) = \mathrm{Var}\left(\mathbb{E}\left[h_n(\omega_n) \mid \omega_n^\varepsilon\right]\right)$$

but rather

$$\mathrm{Cov}(h_n(\omega_n), h_n(\omega_n^\varepsilon)) = \mathrm{Var}\left(\mathbb{E}\left[h_n(\omega_n) \mid \omega_n^{\varepsilon'}\right]\right)$$

where ε' satisfies $(1-\varepsilon')^2 = 1-\varepsilon$.

Moreover we do not have

$$\mathbb{E}\left[g_n(\omega_n) \mid \omega_n^\varepsilon\right] \overset{(d)}{=} \mathbb{P}_{1/2}^{\mathcal{G}_n^\varepsilon}\left[\mathrm{Cross}^\varepsilon(n,n)\right]$$

but rather

$$\mathbb{E}\left[g_n(\omega_n) \mid \omega_n^\varepsilon,\, I^\varepsilon\right] \overset{(d)}{=} \mathbb{P}_{1/2}^{\mathcal{G}_n^\varepsilon}\left[\mathrm{Cross}^\varepsilon(n,n)\right]$$

where I^ε is the set of bits which are resampled. However, the final conclusion of this paragraph is true since we actually have (for instance by using the Fourier decomposition of g_n)

$$\mathrm{Cov}(g_n(\omega_n), g_n(\omega_n^\varepsilon)) = \mathrm{Var}\left(\mathbb{E}\left[g_n(\omega_n) \mid \omega_n^\varepsilon,\, I^\varepsilon\right]\right) = \mathrm{Var}\left(\mathbb{P}_{1/2}^{\mathcal{G}_n^\varepsilon}\left[\mathrm{Cross}^\varepsilon(n,n)\right]\right).$$

2 Chapter 3

Proposition 3.6 should rather be:

Proposition 1. *There exist absolute constants $c_1, c_2 > 0$ such that, for each $R > 0$, monotonic event $A \in \mathcal{F}_R$, and $t \in \mathbb{R}$, if*

$$\frac{c_1 R\,|t|}{\inf\{|\rho(x)| : x \in \mathrm{Ann}_{c_2/R, 2c_2/R}\}} \leq 1,$$

then

$$\left|\mathbb{P}\left[\{f \in A\}\right] - \mathbb{P}[\{f - t \in A\}]\right| \leq \frac{c_1 R\,|t|}{\inf\{|\rho(x)| : x \in \mathrm{Ann}_{c_2/R, 2c_2/R}\}} \sqrt{\mathbb{P}\left[f \in A\right]}.$$

In particular, if $\gamma \geq 0$ is such that $\rho(x) > c_3|x|^\gamma$ for a constant $c_3 > 0$ and sufficiently small $|x|$, then there exist $c, R_0 > 0$ such that, for every $R > R_0$, monotonic event $A \in \mathcal{F}_R$, and $t \in \mathbb{R}$,

$$|\mathbb{P}\left[\{f \in A\}\right] - \mathbb{P}[\{f - t \in A\}]| \leq cR^{1+\gamma}|t|\sqrt{\mathbb{P}\left[f \in A\right]} \text{ if } cR^{1+\gamma}|t| \leq 1.$$

(The "≤ 1" can replaced by "$\leq C$" for any fixed constant C.) The mistake comes from Corollary 3.10, which should rather be:

Corollary 2. *There exists $c_3 > 0$ such that, for every $h \in H$ satisfying $||h||_H \leq 1$ and $A \in \mathcal{F}$,*

$$|\mathbb{P}[f \in A] - \mathbb{P}[f - h \in A]| \leq c\|h\|_H\sqrt{\mathbb{P}\left[f \in A\right]}.$$

This is not at all a problem for the rest of the proof. The consequence is only that in Corollary 3.7 we have to assume that $cR\varepsilon \leq 1$ and we have to make the analogous assumptions in the results of Section 4. But we apply these results in the rest of the paper only for $cR\varepsilon$ (or the analogous quantities) very small.

Actually, the reason why we need to make these further assumptions is that we want to have the term $\sqrt{\mathbb{P}\left[f \in A\right]}$ on the right-hand-side of the inequalities. This is very important for our bootstraps in Section 6.

In a new version of this work ([MV]), we rather use the following less quantitative result:

Proposition 3. *There exist absolute constants $c_1, c_2 > 0$ such that, for each $R > 0$, monotonic event $A \in \mathcal{F}_R$, and $t \in \mathbb{R}$,*

$$|\mathbb{P}\left[\{f \in A\}\right] - \mathbb{P}[\{f - t \in A\}]| \leq \frac{c_1 R\,|t|}{\inf\{|\rho(x)| : x \in \mathrm{Ann}_{c_2/R, 2c_2/R}\}}.$$

In particular, if $\gamma \geq 0$ is such that $\rho(x) > c_3|x|^\gamma$ for a constant $c_3 > 0$ and sufficiently small $|x|$, then there exist $c, R_0 > 0$ such that, for every $R > R_0$, monotonic event $A \in \mathcal{F}_R$, and $t \in \mathbb{R}$,

$$|\mathbb{P}\left[\{f \in A\}\right] - \mathbb{P}[\{f - t \in A\}]| \leq cR^{1+\gamma}|t|.$$

In [MV], we replace the use of the $\sqrt{\mathbb{P}\left[f \in A\right]}$ by a "sprinkling quasi-independence" property (see Proposition 6.2 therein). This enables us to strengthen our sharpness result (i.e. we obtain exponential decay of connexion properties even when q decays polynomially fast rather than the $\leq \exp(-c\log^2(s))$ estimate which is in my thesis).

3 Chapter 6

There is a (small) mistake at the end of the proof of Proposition 2.1. The reader can rather read the corresponding paper [Vanb] (where other corrections have also been made).

4 Chapter 7

There are (not very important) mistakes in the proofs of Section 3.3. The reader can rather read the corresponding paper [Vana] (where other corrections have also been made).

References

[MV] Stephen Muirhead and Hugo Vanneuville. The sharp phase transition for level set percolation of smooth planar Gaussian fields.

[Vana] Hugo Vanneuville. The annealed spectral sample of Voronoi percolation.

[Vanb] Hugo Vanneuville. Quantitative quenched Voronoi percolation and applications.